# DEVELOPMENT OF MITTAG-LEFFLER FUNCTION OF FRACTIONAL DIFFERENTIAL OPERATORS

**Article** · January 2023

DOI: 10.17605/OSF.IO/U3TA7

**Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)**
ISSN: 1671-5497
E-Publication: Online Open Access
Vol: 41 Issue: 12-2022
DOI 10.17605/OSF.IO/U3TA7

# DEVELOPMENT OF MITTAG-LEFFLER FUNCTION OF FRACTIONAL DIFFERENTIAL OPERATORS

## A. K. THAKUR

Department of Mathematics, Dr. C.V. Raman University, Bilaspur (C.G.)
Email: drakthakurmath@gmail.com

## S. K. SAHANI

Department of Mathematics, MIT Campus, (T.U.), Janakpurdham, Nepal.
Corresponding Author Email: sureshkumarsahani35@gmail.com

## SUREKHA DEWANGAN

Department of Mathematics, Research Scholar, Dr. C.V. Raman University, Bilaspur (C.G.)
Email: sure.dewap@gmail.com

**Abstract:**

Develop the Mittag-Leffler function of Fractional Differential operators with the Appell's function as a kernel. Also develop some new type of fractional integral equations of Mittag-Leffler as special case.

## INTRODUCTION

In the year 1903, the Swedish mathematician Mittag-Leffler was given $E_\alpha(z)$ in form

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+\beta)} \ (\alpha \in \mathbb{C};\ \Re(\alpha) > 0) \quad (1.1)$$

In the year 1905, Wiman A. (1905) introduced the generalization of $E_\alpha(z)$ in the form

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+\beta)} \ (\alpha, \beta \in \mathbb{C};\ \Re(\alpha), \Re(\beta) > 0) \quad (1.2)$$

Which is called the generalized Mittag- Leffler function.

In the year 1971, Prabhakar T. R. The generalization of $E_\alpha(z)$ in the form

Mittag-Leffler function $E^{\gamma}_{\propto,\beta}(z)$ as

$$E^{\gamma}_{\propto,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\alpha n+\beta)} \frac{z^n}{n!}\ , \quad (1.3)$$

Where $(\gamma)_n$ denotes the Pochhammer symbol provided by

$$(\gamma)_n = \gamma(\gamma+1)(\gamma+2)\ldots(\gamma+n-1) = \frac{\Gamma(\gamma+n)}{\Gamma(\gamma)}$$

And $\alpha, \beta, \gamma \in \mathbb{C};\ \Re(\alpha) > 0, \Re(\beta) > 0.$

By the same fashion in the year 2012, Dorrego and Cerutti gave us another generalization of Mittag-Leffler function $E^{\gamma}_{k,\propto,\beta}(z)$ as

**Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)**
ISSN: 1671-5497
E-Publication: Online Open Access
Vol: 41 Issue: 12-2022
DOI 10.17605/OSF.IO/U3TA7

$$E^{\gamma}_{k,\propto,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{n,k}}{\Gamma_k(\alpha n+)} \frac{z^n}{n!} \,, \tag{1.4}$$

Where $\alpha, \beta, \gamma \in \mathbb{C}$; $\Re(\alpha), \Re(\beta)$ , $k > 0$ and $(\gamma)_{n,k}$ is k-Pochhammer symbol provided by Diaz and Pariguan (2012) as

$$(\gamma)_n = \gamma(\gamma + k)(\gamma + 2k) \ldots (\gamma + (n - 1)k) = \frac{\Gamma_k(\gamma + nk)}{\Gamma_k(\gamma)}$$

And $\Gamma_k$ the k – Gamma function given by

$$\Gamma_k(z) = \int_0^{\infty} t^{z-1} e^{-\frac{t^k}{k}} dt \,, \Re(\alpha) > 0.$$

Which is also known as k- Mittag- Leffler function.

In the year 2017 Cerutti et. al, generalized the Mittag – Leffler (p-k) function $pE^{\gamma}_{k,\alpha,\beta}(z)$ was defined as

$$pE^{\gamma}_{k,\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{p(\gamma)_{n.k}}{p^{\Gamma_k(\alpha n+\beta)}} \frac{z^n}{n!} \tag{1.5}$$

Where $\alpha, \beta, \gamma, z \in \mathbb{C}$; $\Re(\alpha), \Re(\beta),, \Re(\gamma) > 0$ ; $p, k \in \mathbb{R}^+ \backslash \{0\}$ and $p^{(\gamma)_{n,k}}$ is the Pochhammer (p-k)-sympol given by Gehlot K. S. (2017) define as

$$p^{(\gamma)_{n,k}} = \left(\frac{\gamma p}{k}\right)\left(\frac{\gamma p}{k} + p\right)\left(\frac{\gamma p}{k} + 2p\right) \ldots\ldots \left(\frac{\gamma p}{k} + (n\ - 1\ )p\right) = \frac{p^{\Gamma_k}(\gamma + nk)}{p^{\Gamma_k}(\gamma)}$$

And $p^{\Gamma_k(z)}$ is defined as

$$p^{\Gamma_k}(z) = \int_0^{\infty} t^{z-1} e^{-\frac{t^k}{p}} \ dt, \qquad (z \in \mathbb{C} \backslash k\mathbb{Z}^-; p, k \in \mathbb{R}^+ \backslash \{0\})$$

In the year 2020, Ayub et. al. introduced one more generalization of equation (1.5) called the Mittag-Leffler (p,s,k) function as

$$pE^{\gamma,s}_{k,\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{p^{(\gamma)_{n,k,s}}}{p^{\Gamma_{s,k}(\alpha n+\beta)}} \frac{z^n}{n!} \tag{1.6}$$

Where $k, p \in \mathbb{R}, \alpha, \beta, \gamma \ \in \mathbb{C}$, $\Re e(\alpha) > 0$, $\Re e(\beta)$ , $\Re e(\gamma) > 0$ and $p^{(\gamma)_{n,k,s}}$ is the Pochhammer (p, s, k)-symbol given by Gehlot and Nantomah (2018) as

$$p^{(\gamma)_{n,k,s}} = \left[\frac{\gamma p}{k}\right]_s \left[\frac{\gamma p}{k} + p\right]_s \ldots \left[\frac{\gamma p}{k} + (n - 1)p\right]_s = \prod_{i=0}^{n-1} \left[\frac{\gamma p}{k} + ip\right]_s.$$

Where $[\gamma]_s = \frac{1-s^{\gamma}}{1-s}, \quad \forall \gamma \in \mathbb{R}, 0 < s < 1$ .

And $p^{\Gamma_{s,k}}$ gamma (p, s, k)-function in the form of

$$p^{\Gamma_{s,k}(\xi)} = \frac{s}{k} \lim_{n\to\infty} \frac{n!\, p^{n+1}(snp)^{\frac{\xi}{k}-1}}{p^{(\xi)_{n,k}}}$$

These following well-known definitions and results are essential for the study:

**Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)**
ISSN: 1671-5497
E-Publication: Online Open Access
Vol: 41 Issue: 12-2022
DOI 10.17605/OSF.IO/U3TA7

**Definition (1):**

Generalization of Wright hypergeometric function ${}_p\Psi_q$ [z]

$$pE_{k,\theta,\vartheta}^{\varrho,s}(z)=\frac{k\left(\frac{sp}{k}\right)^{-\vartheta/k}}{\Gamma\left(\frac{\rho}{k}\right)}\,{}_1\Psi_1\left[z(sp)^{1-\frac{\theta}{k}}\left|\begin{matrix}\left(\frac{\rho}{k},1\right)\\\left(\frac{\vartheta}{k},\frac{\theta}{k}\right)\end{matrix}\right.\right] \qquad (1.7)$$

**Definition (2):**

Generalization of Fox H-function

$$pE_{k,\theta,\vartheta}^{\varrho,s}(z)=\frac{k\left(\frac{sp}{k}\right)^{-\vartheta/k}}{\Gamma\left(\frac{\rho}{k}\right)}\,H_{1,2}^{1,1}\left[-z(sp)^{1-\frac{\theta}{k}}\left|\begin{matrix}\left(1-\frac{\rho}{k},1\right)\\(0,1),\left(1-\frac{\vartheta}{k},\frac{\theta}{k}\right)\end{matrix}\right.\right] \qquad (1.8)$$

In the year 1935, Wright E. generalized hypergeometric function $p\Psi q\ [z]$ in the form

$$p\Psi q\ [z]=p\Psi q\left[\begin{matrix}(a_1,\alpha_1),\dots,(a_p,\alpha_p)\\(b_1,\beta_1),\dots,(b_q,\beta_q)\end{matrix};z\right]$$

$$=\sum_{n=0}^{\infty}\frac{\Gamma(a_1+\alpha_1 n)\dots\Gamma(a_p+\alpha_p n)}{\Gamma(b_1+\beta_1 n)\dots\Gamma(b_q+\beta_q n)}\,\frac{z^n}{n!} \qquad (1.9)$$

$$=H_{p,q+1}^{1,p}\left[-z\left|\begin{matrix}(1-a_1,\alpha_1),\dots\dots\dots\dots,(1-a_p,\quad\alpha_p)\\(0,1),(1-b_1,\beta_1),\dots\dots,(1-b_q,\ \beta_q)\end{matrix}\right.\right]$$

Where $z\in C,\ a_i,b_j\in\ \mathbb{R}^+$ and $\alpha_i,\beta_j\in\mathbb{R}^+\ ,\alpha_i,\beta_j\neq 0\,;i=1,2,\dots,p\,;j=1,2,\dots,q.$ .

The generalized fractional integral operator introduced by Saigo and Maeda associated by the appell function $F_3$ in the form

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}f\right)(x)$$

$$=\frac{x^{-\alpha}}{\Gamma(\gamma)}\int_0^x(x-t)^{\gamma-1}\,t^{-\alpha'}F_3\left(\alpha,\alpha';\beta,-\beta';\gamma,1-\frac{t}{x},1-\frac{x}{t}\right)f(t)dt \qquad (1.10)$$

Where $(\mathcal{R}e(\gamma)>0\,)$

And

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}f\right)(x)$$

$$=\frac{x^{-\alpha'}}{\Gamma(\gamma)}\int_x^\infty(x-t)^{\gamma-1}t^{-\alpha}\,F_3\left(\alpha,\alpha';\beta,-\beta';\gamma,1-\frac{t}{x},1-\frac{x}{t}\right)f(t)dt \qquad (1.11)$$

$where(\mathcal{R}e(\gamma)>0\,)$

Where $\alpha,\alpha',\beta,\beta',\gamma,\in\mathbb{C}$ and $x>\ 0$.

In the year 1998, Saigo and Maeda define fractional differential operators in the form

$$\left(D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}f\right)(x)=\left(I_{0+}^{-\alpha',-\alpha,\,\beta',-\beta,-\gamma}f\right)(x) \qquad (1.12)$$

**Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)**
ISSN: 1671-5497
E-Publication: Online Open Access
Vol: 41 Issue: 12-2022
DOI 10.17605/OSF.IO/U3TA7

$= \left(\frac{d}{dx}\right)^m \left(I_{0+}^{-\alpha',-\alpha,\beta'+m,-\beta,-\gamma+m} f\right)(x)$ $\left(D_{-}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x) =$
$\left(I_{-}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma} f\right)(x)$ (1.13)

$$= \left(-\frac{d}{dx}\right)^m \left(I_{-}^{-\alpha',-\alpha,-\beta',-\beta+m,-\gamma+m} f\right)(x)$$

Where $(R(\gamma) > 0; m = [R(\gamma)] + 1)$.

We obtain a corresponding relationship if we take $\alpha = \alpha + \beta, \alpha' = \beta' = 0,\ \beta = -\eta$ and $\gamma = \alpha$ in above equations

$$(I_{0+}^{\alpha+\beta,0,-\eta,0,\alpha} f)(x) = \left(I_{0+}^{\alpha,\beta,\eta} f\right)(x) \quad (1.14)$$

And $(I_{-}^{\alpha+\beta,0,-\eta,0,\alpha} f)(x) = \left(I_{-}^{\alpha,\beta,\eta} f\right)(x)$ (1.15)

In the year 1978 Saigo define fractional integral operator and is denoted by $I_{0+}^{\alpha,\beta,\eta}$

$$(D_{0+}^{\alpha+\beta,0,-\eta,0,\alpha} f)(x) = \left(D_{0+}^{\alpha,\beta,\eta} f\right)(x) \quad (2.1.16)$$

$$(D_{-}^{\alpha+\beta,0,-\eta,0,\alpha} f)(x) = \left(D_{-}^{\alpha,\beta,\eta} f\right)(x) \quad (1.17)$$

**Corollary 1.** In the year 1998, Saigo M. and Maeda N. Let $\alpha, \alpha', \beta, \beta', \gamma, \rho, \in \mathbb{C}$ be such that R ($\gamma$) > 0 and R ($\rho$) > max {0, R ($\alpha' - \beta'$), R($\alpha + \alpha' + \beta - \gamma$)} then we get

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} x^{\rho-1}\right)(x) = \frac{\Gamma(\rho)\Gamma(\rho-\alpha'+\beta')\Gamma(\rho+\gamma-\alpha-\alpha'-\beta)}{\Gamma(\rho+\beta')\Gamma(\rho+\gamma-\alpha-\alpha')\Gamma(\rho+\gamma-\alpha'-\beta)} x^{\rho-\alpha-\alpha'+\gamma-1} \quad (1.18)$$

**Corollary 2.** In 1998, Saigo M. and Maeda N .Let $\alpha, \alpha', \beta, \beta', \gamma, \rho, \in \mathbb{C}$ be such that R($\gamma$) > 0 and R($\rho$) < 1 + min { R ($-\beta$), R ($\alpha + \alpha' - \gamma$), R ($\alpha + \beta' - \gamma$)} then we get

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma} x^{\rho-1}\right)(x) = \frac{\Gamma(1+\alpha+\alpha'-\gamma-\rho)\Gamma(1+\alpha+\beta'-\gamma-\rho)\Gamma(1-\beta-\rho)}{\Gamma(1-\rho)\Gamma(1+\alpha+\alpha'+\beta'-\gamma-\rho)\Gamma(1+\alpha-\beta-\rho)} x^{\rho-\alpha-\alpha'+\gamma-1} \quad (1.19)$$

## 2. Main Results

Here we develop some results of Mittag-Leffler function according to the Saigo-Maeda fractional integral equations of generalized Wright function

**Theorem 1.** Let $p, p', p'', q, q', q'', \gamma, \rho, \theta_i, \mu_i, \vartheta_i, \sigma_i \in C$ with $\Re(\sigma_i), \Re(\theta_i), \Re(\vartheta_i), \Re(\mu_i), \Re(\gamma) > 0$ $and$ $\Re(\rho) > max\{0, \Re(\alpha' - \beta'), \Re(\alpha + \alpha' + \beta - \gamma)\}$ and t > 0, $p_i, q_i\, k_i, s_i > 0, \forall\, i = 1,2, \ldots, r$, then left sided fractional integral formula holds

$$\left(I_{0+}^{\alpha,\alpha',\alpha'',\beta,\beta',\beta'',\gamma} t^{\rho-1} \left\{\prod_{i=1}^{r} p_i E_{k_i,\theta_i,\vartheta_{i,}\mu_i}^{\sigma_i,r_i}(t)\right\}\right)(t) = t^{\rho+\gamma-\alpha-\alpha'-\alpha''-1} \prod_{i=1}^{r} \left\{\frac{k_i (r_i p_i q_i)^{-\vartheta_i/k_i}}{\Gamma\left(\frac{\sigma_i}{k_i}\right)}\right.$$

$$\left.\times\ {}_{r+4}\Psi_{r+4}\left[\left.\begin{matrix}\left(\frac{\sigma_i}{k_i},1\right)_{1,r},(\rho,r),(\rho-\alpha'-\alpha''+\beta''+\beta',r),(\rho+\gamma-\alpha-\alpha'-\alpha''-\beta,r)\\ \left(\frac{\vartheta_i}{k_i},\frac{\theta_i\mu_i}{k_i\,k_i}\right)_{1,r},(\rho+\beta'+\beta'',r),(\rho+\gamma-\alpha-\alpha',r),(\rho+\gamma-\alpha'-\alpha''-\beta,r)\end{matrix}\right|\ (r_i p_i q_i)^{\left(1-\frac{\theta_i}{k_i}\right)} t^r\right]\right\} \quad (2.1)$$

**Proof.** We have by the order of integration and summation of Mittag-Leffler function of (1.6), we get (say $F_1$)

**Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)**
ISSN: 1671-5497
E-Publication: Online Open Access
Vol: 41 Issue: 12-2022
DOI 10.17605/OSF.IO/U3TA7

$$(F_1) = \prod_{i=1}^{r}\left\{\sum_{n_i=0}^{\infty}\frac{{p_i}^{(\sigma_i)}{}_{n_i,k_i,r_i}}{{p_i}^{\Gamma_{r_i,k_i}(n_i\theta_i+\vartheta_i)}}\frac{1}{n_i!}\right\}\left(I_{0+}^{\alpha,\alpha',\alpha'',,\beta,\beta',\beta'',\gamma}t^{n_ir+\rho+q-1}\right)(t) \quad (2.2)$$

Now, with the help of result (1.19) into (2.2) to obtain

$$(F_1) = \prod_{i=1}^{r}\left\{\sum_{n_i=0}^{\infty}\frac{{p_i}^{(\sigma_i)}{}_{n_i,k_i,r_i}}{{p_i}^{\Gamma_{r_i,k_i}(n_i\theta_i+\vartheta_i+\mu_i)}}\frac{1}{n_i!}\frac{\Gamma(\rho+n_ir)\Gamma(\rho+n_ir-\alpha'-\alpha''+\beta'+\beta'')}{\Gamma(\rho+n_ir+\beta'+\beta'')\Gamma(\rho+n_ir+\gamma-\alpha-\alpha'-\alpha'')}\right.$$
$$\left.\times\frac{\Gamma(\rho+n_ir+\gamma-\alpha-\alpha'-\alpha''-\beta)}{\Gamma(\rho+n_ir+\gamma-\alpha'-\alpha''-\beta)}t^{\rho+n_ir+\gamma-\alpha-\alpha'-\alpha''-1}\right\} \quad (2.3)$$

By includind the result of (1.7) and (1.8) into (2.3), we get

$$(F_1) = t^{\rho+\gamma-\alpha-\alpha'-\alpha''-1}\prod_{i=1}^{r}\left\{\frac{k_i(r_ip_iq_i)^{-\vartheta_i/k_i}}{\Gamma\left(\frac{\sigma_i}{k_i}\right)}\sum_{n_i=0}^{\infty}\frac{1}{n_i!}\frac{\Gamma\left(\frac{\sigma_i}{k_i}+n_i\right)}{\Gamma((n_i\theta_i+\vartheta_i+\mu_i)/k_i)}\frac{\Gamma(\rho+n_ir)}{\Gamma(\rho+n_ir+\beta'+\beta'')}\right.$$
$$\left.\times\frac{\Gamma(\rho+n_ir-\alpha'-\alpha''+\beta'+\beta'')\Gamma(\rho+n_ir+\gamma-\alpha-\alpha'-\alpha''-\beta)}{\Gamma(\rho+n_ir+\gamma-\alpha-\alpha'-\alpha'')\Gamma(\rho+n_ir+\gamma-\alpha'-\alpha''-\beta)}\left((r_ip_iq_i)^{\left(1-\frac{\theta_i}{k_i}\right)}t^r\right)^{n_i}\right\} \quad (2.4)$$

Finally, we obtained at the right hand side of result (2.2) after the re-insertion of the above series, which was in the form of generalized Wright hyperactive geometric function defined by (1.11).

**Corollary 1.** By equation (1.10), then we get the Fox H-function:

$$\left(I_{0+}^{\alpha,\alpha',\alpha'',\beta,\beta',\beta'',\gamma}t^{\rho-1}\left\{\prod_{i=1}^{r}p_iE_{k_i,\theta_i,\vartheta_{i,},\mu_i}^{\sigma_i,s_i}(t)\right\}\right)(t) = t^{\rho+\gamma-\alpha-\alpha''-\alpha'-1}\prod_{i=1}^{r}\left\{\frac{k_i(r_ip_iq_i)^{-\vartheta_i/k_i}}{\Gamma\left(\frac{\sigma_i}{k_i}\right)}\right.$$
$$\left.\times H_{r+4,r+5}^{1,r+4}\left[-(r_ip_iq_i)^{\left(1-\frac{\theta_i}{k_i}\right)}t^r\,\middle|\,\begin{matrix}\left(1-\frac{\sigma_i}{k_i},1\right)_{1,r},(1-\rho,r),(1-\rho+\alpha'+\alpha''-\beta',\beta'',r),(1-\rho-\gamma+\alpha+\alpha'+\alpha''+\beta,r)\\(0,1),\left(1-\frac{\vartheta_i}{k_i},\frac{\theta_i}{k_i},\frac{\mu_i}{k_i}\right)_{1,r},(1-\rho-\beta'+\beta''+,r),(1-\rho-\gamma+\alpha+\alpha'+\alpha'',r),(1-\rho-\gamma+\alpha'+\alpha''+\beta,r)\end{matrix}\right]\right\}$$

.**Theorem 2.** Let $\alpha,\alpha',\alpha'',\beta,\beta',\beta''\gamma,\rho,\theta_i,\vartheta_i,\mu_i,\sigma_i\in\mathbb{C}$ with $\Re(\sigma_i),\Re(\theta_i),\Re(\mu_i),\Re(\nu_i),\Re(\mu_i),\Re(\gamma)>0$, $\Re(\rho)<1+min\{\Re(-\beta),\Re(\alpha+\alpha'+\alpha'',-\gamma),\Re(\alpha-\beta'-\beta''-\gamma)\}$ and $t,p_i,q_i,k_i,r_i>0$, $\forall\ i=1,2,\ldots,r$, then the right sided fractional integral formula holds

$$\left(I_{-}^{\alpha,\alpha',\alpha'',,\beta,\beta',\beta'',\gamma}t^{\rho-1}\left\{\prod_{i=1}^{r}p_iE_{k_i,\theta_i,\vartheta_{i,},\mu_i}^{\sigma_i,r_i}\left(\frac{1}{t}\right)\right\}\right)(t) = t^{\rho-\alpha-\alpha'+\alpha''+\gamma-1}\prod_{i=1}^{r}\left\{\frac{k_i(r_ip_iq_i)^{\frac{-\vartheta_i}{k_i}}}{\Gamma\left(\frac{\sigma_i}{k_i}\right)}\right.$$

**Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)**
ISSN: 1671-5497
E-Publication: Online Open Access
Vol: 41 Issue: 12-2022
DOI 10.17605/OSF.IO/U3TA7

$$\times \ {}_{r+4}\Psi_{r+4}\left[\begin{matrix}\left(\frac{\sigma_i}{k_i},1\right)_{1,r},(1-\rho-\beta,r),(1-\rho+\alpha+\alpha'-\gamma,r),(1-\rho+\alpha+\beta'+\beta''-\gamma,r)\\ \left(\frac{\vartheta_i}{k_i},\frac{\theta_i}{k_i},\frac{\mu_i}{k_i}\right)_{1,r},(1-\rho,r),(1-\rho+\alpha+\alpha'+\beta'+\beta''-\gamma,r),(1-\rho+\alpha-\beta,r)\end{matrix}\middle|\frac{(r_ip_iq_i)^{\left(1-\frac{\theta_i}{k_i}\right)}}{t^r}\right]\Bigg\} \qquad (2.5)$$

**Proof.** We have by the order of integration and summation of Mittag-Leffler function it is define as

$$F_2=\prod_{i=1}^{r}\left\{\sum_{n_i=0}^{\infty}\frac{{p_i}^{(\sigma_i)}{}_{n_i,k_i,r_i}}{{p_i}^{\Gamma_{s_i,k_i}}(n_i\theta_i+\vartheta_i+,\mu_i)}\frac{1}{n_i!}\right\}\left(I_{-}^{\alpha,\alpha',\alpha'',\beta,\beta',\beta'',\gamma}t^{\rho-n_ir-1}\right)(t) \qquad (2.6)$$

Now, with the help of result (1.19) into (2.6) to obtain

$$F_2=\prod_{i=1}^{r}\left\{\sum_{n_i=0}^{\infty}\frac{{p_i}^{(\sigma_i)}{}_{n_i,k_i,r_i}}{{p_i}^{\Gamma_{r_i,k_i}}(n_i\theta_i+\vartheta_i+,\mu_i)}\frac{1}{n_i!}\right\}\frac{\Gamma(1-\rho-\beta+n_ir)\Gamma(1-\rho+n_ir-\gamma+\alpha+\beta'+\beta'')}{\Gamma(1-\rho+n_ir)\Gamma(1-\rho+n_ir-\gamma+\alpha+\alpha'+\alpha'')}$$
$$\times\frac{\Gamma(1-\rho+n_ir-\gamma+\alpha+\alpha'+\alpha'')}{\Gamma(1-\rho+n_ir+\alpha-\beta)}t^{\rho+n_ir-\alpha-\alpha'+\alpha''+\gamma-1} \qquad (2.7)$$

By including the result of (1.7) and (1.8) into (2.7), we get

$$F_2=t^{\rho+\gamma-\alpha-\alpha'-\alpha''-1}\prod_{i=1}^{r}\left\{\frac{k_i(s_ip_i)^{-\vartheta_i/k_i}}{\Gamma\left(\frac{\sigma_i}{k_i}\right)}\sum_{n_i=0}^{\infty}\frac{1}{n_i!}\frac{\Gamma\left(\frac{\sigma_i}{k_i}+n_i\right)}{\Gamma((n_i\theta_i+\vartheta_i)/k_i)}\frac{\Gamma(1-\rho-\beta+n_ir)}{\Gamma(1-\rho+n_ir)}\right.$$
$$\left.\times\frac{\Gamma(1-\rho+n_ir-\gamma+\alpha+\beta'+\beta'')\Gamma(1-\rho+n_ir-\gamma+\alpha+\alpha'+\alpha'')}{\Gamma(1-\rho+n_ir-\gamma+\alpha+\alpha'+\alpha'')\Gamma(1-\rho+n_ir+\alpha-\beta)}\left(\frac{(r_ip_iq_i)^{\left(1-\frac{\theta_i}{k_i}\right)}}{t^r}\right)^{n_i}\right\}$$

Finally, we obtained at the right hand side of result (2.5) after the re-insertion of the above series, which was in the form of generalized Wright hypergeometric function defined by (1.11).

C**orollary 2.** By equation (1.10), then we get a new result in terms of Fox H-function:

$$\left(I_{-}^{\alpha,\alpha',\alpha'',\beta,\beta',\beta'',\gamma}t^{\rho-1}\left\{\prod_{i=1}^{r}p_iE_{k_i,\theta_i,\vartheta_i,,\mu_i}^{\sigma_i,s_i}\left(\frac{1}{t}\right)\right\}\right)(t)=$$
$$t^{\rho-\alpha-\alpha'-\alpha''+\gamma-1}\prod_{i=1}^{r}\left\{\frac{k_i(r_ip_iq_i)^{\frac{-\vartheta_i}{k_i}}}{\Gamma\left(\frac{\sigma_i}{k_i}\right)}H_{r+4,r+5}^{1,r+4}\left[\begin{matrix}\left(1-\frac{\sigma_i}{k_i},1\right)_{1,r},(\rho+\beta,r),(\rho-\alpha-\beta'-\beta''+\gamma,r),(\rho-\alpha-\alpha'+\gamma,r)\\(0,1),\left(1-\frac{\vartheta_i}{k_i},\frac{\theta_i}{k_i}\frac{\mu_i}{k_i}\right)_{1,r},(\rho,r)\ (\rho+\gamma-\alpha-\alpha'-\alpha'',r),(\rho-\alpha+\beta,r)\end{matrix}\middle|\frac{-(r_ip_iq_i)^{\left(1-\frac{\theta_i}{k_i}\right)}}{t^r}\right]\right\}$$

## 3. Development of Some results of Mittag-Leffler function of Fractional Differential operators

**Theorem 1.** Let $\alpha,\alpha',\alpha'',\beta,\beta',\beta'',\gamma,\rho,\theta_i,\vartheta_i,\sigma_i,,\mu_i\in\mathbb{C}$ with $\Re(\sigma_i),\Re(\theta_i),\Re(\nu_i),\Re(\gamma),\Re(\gamma),\Re(\mu_i)>0$ , $\Re(\rho)>max\{0,\Re(\beta-\alpha),\Re(\gamma-\alpha-\alpha'-\alpha''-\beta'-\beta'')\}$ and t, $p_i,q_ik_i,r_i>0,\forall\ i=1,2,\ldots,r$, then left sided fractional differential formula holds

**Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)**
ISSN: 1671-5497
E-Publication: Online Open Access
Vol: 41 Issue: 12-2022
DOI 10.17605/OSF.IO/U3TA7

$$\left(D_{0+}^{\alpha,\alpha'\alpha'',,\beta,\beta'\beta'',\gamma}t^{\rho-1}\left\{\prod_{i=1}^{r}p_i E_{k_i,\theta_i,\vartheta_{i,},\mu_i}^{\sigma_i,s_i}(t)\right\}\right)(t)=t^{\rho+\alpha+\alpha'+\alpha''-\gamma-1}\prod_{i=1}^{r}\left\{\frac{k_i(s_ip_i)^{-\frac{\vartheta_i}{k_i}}}{\Gamma\left(\frac{\sigma_i}{k_i}\right)}\right.$$

$$\times\ {}_{r+3}\Psi_{r+3}\left[\left.\begin{matrix}\left(\frac{\sigma_i}{k_i},1\right)_{1,r},(\rho,r)\ (\rho+\alpha-\beta,r)\ (\rho+\alpha+\alpha'+\alpha''+\beta'+\beta''-\gamma,r)\\ \left(\frac{\vartheta_i}{k_i},\frac{\theta_i}{k_i},\frac{\mu_i}{k_i}\right)_{1,r},(\rho-\beta,r)(\rho+\alpha+\alpha'+\alpha''-\gamma,r)(\rho+\alpha+\beta'+\beta''-\gamma,r)\end{matrix}\right|(s_ip_i)^{\left(1-\frac{\theta_i}{k_i}\right)}\ t^r\right]\Bigg\}\qquad(3.1)$$

**Proof.** By utilizing equation (1.6), we get (say $F$)

$$F_3=\prod_{i=1}^{r}\left\{\sum_{n_i=0}^{\infty}\frac{p_i{}^{(\sigma_i)}{}_{n_i,k_i,s_i}}{p_i{}^{\Gamma_{s_i,k_i}(n_i\theta_i+\vartheta_i)}}\frac{1}{n_i!}\right\}\left(D_{0+}^{\alpha,\alpha',\alpha'',\beta,\beta',\beta'',\gamma}t^{n_ir+\rho-1}\right)(t)\qquad(3.2)$$

Now, with the help of result (1.19) into (3.2) to obtain

$$F_3=\prod_{i=1}^{r}\left\{\sum_{n_i=0}^{\infty}\frac{p_i{}^{(\sigma_i)}{}_{n_i,k_i,s_i}}{p_i{}^{\Gamma_{s_i,k_i}(n_i\theta_i+\vartheta_i)}}\frac{1}{n_i!}\right\}\frac{\Gamma(\rho+n_ir)\Gamma(\rho+n_ir+\alpha-\beta)}{\Gamma(\rho+n_ir-\beta)\Gamma(\rho+n_ir+\alpha+\alpha'+\alpha''-\gamma)}$$

$$\times\frac{\Gamma(\rho+n_ir-\gamma+\alpha+\alpha'+\alpha''+\beta'+\beta'')}{\Gamma(\rho+n_ir+\alpha+\alpha'+\beta'+\beta''-\gamma)}t^{\rho+n_ir+\alpha++\alpha'+\alpha''-\gamma-1}\qquad(3.3)$$

By including the result of (1.7) and (1.8) into (3.3), we get

$$F_3=t^{\rho-\gamma+\alpha+\alpha''+\alpha'-1}\prod_{i=1}^{r}\left\{\frac{k_i(s_ip_i)^{-\vartheta_i/k_i}}{\Gamma\left(\frac{\sigma_i}{k_i}\right)}\sum_{n_i=0}^{\infty}\frac{1}{n_i!}\frac{\Gamma\left(\frac{\sigma_i}{k_i}+n_i\right)}{\Gamma((n_i\theta_i+\vartheta_i)/k_i)}\frac{\Gamma(\rho+n_ir)}{(\rho+n_ir-\beta)}\right.$$

$$\times\frac{\Gamma(\rho+n_ir+\alpha+\alpha'+\alpha''+\beta'-\beta''-\gamma)\Gamma(\rho+n_ir+\alpha-\beta)}{\Gamma(\rho+n_ir+\alpha+\alpha'+\alpha''-\gamma)\Gamma(\rho+n_ir+\alpha+\alpha'-\beta'-\beta''-\gamma)}\left.\left((s_ip_i)^{\left(1-\frac{\theta_i}{k_i}\right)}t^r\right)^{n_i}\right\}$$

At last, we obtained at the right hand side of result (3.1) after the re-insertion of the above series, which was in the form of generalized Wright hypergeometric function defined by (1.11).

**Corollary 3.** By the light of (1.10), then we get the new result in terms of Fox H-function:

$$\left(D_{0+}^{\alpha,\alpha',\alpha'',\beta,\beta',\beta'',\gamma}t^{\rho-1}\left\{\prod_{i=1}^{r}p_i E_{k_i,\theta_i,\vartheta_i}^{\sigma_i,s_i}(t)\right\}\right)(t)=t^{\rho+\alpha+\alpha''+\alpha'-\gamma-1}\prod_{i=1}^{r}\left\{\frac{k_i(s_ip_i)^{-\frac{\vartheta_i}{k_i}}}{\Gamma\left(\frac{\sigma_i}{k_i}\right)}H_{r+3,r+4}^{1,r+3}\right.$$

$$\left[\left.\begin{matrix}\left(1-\frac{\sigma_i}{k_i},1\right)_{1,r},(1-\rho,r),(1-\rho-\alpha+\beta,r),(1-\rho-\alpha-\alpha'-\alpha''-\beta'-\beta''+\gamma,r)\\ (0,1),\left(1-\frac{\vartheta_i}{k_i},\frac{\theta_i}{k_i},\frac{\mu_i}{k_i}\right)_{1,r},(1-\rho+\beta,r),(1-\rho-\alpha-\alpha'-\alpha''+\gamma,r),(1-\rho-\alpha-\alpha'-\beta'-\beta''+\gamma,r)\end{matrix}\right|-(s_ip_i)^{\left(1-\frac{\theta_i}{k_i}\right)}\ t^r\right]\Bigg\}.$$

**Theorem 2.** Let $\alpha,\alpha',\alpha'',\beta,\beta',\beta'',\gamma,\rho,\theta_i,\vartheta_i,\sigma_i,,\mu_i\in\mathbb{C}$ with $\Re(\sigma_i),\Re(\theta_i),\Re(\nu_i),\Re(\gamma),\Re(\mu_i)>0,\Re(\rho)<1+min\{\Re(\beta'),\ \Re(\gamma-\alpha'-\beta),\ \Re(\gamma-\alpha-\alpha'-\alpha'')\}$ *and* t , $p_i,k_i,s_i>0,\ \forall\ i=1,2,\dots,r$, then right sided fractional differential formula holds

**Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)**
ISSN: 1671-5497
E-Publication: Online Open Access
Vol: 41 Issue: 12-2022
DOI 10.17605/OSF.IO/U3TA7

$$\left(D_{-}^{\alpha,\alpha',\alpha'',\beta,\beta',\beta'',\gamma}t^{\rho-1}\left\{\prod_{i=1}^{r}p_iE_{k_i,\theta_i,\vartheta_i}^{\sigma_i,s_i}\left(\frac{1}{t}\right)\right\}\right)(t)=t^{\rho+\alpha+\alpha''+\alpha'-\gamma-1}\prod_{i=1}^{r}\left\{\frac{k_i(s_ip_i)^{-\frac{\vartheta_i}{k_i}}}{\Gamma\left(\frac{\sigma_i}{k_i}\right)}\right.$$

$$\left.\times\ {}_{r+3}\Psi_{r+3}\left[\begin{matrix}\left(\frac{\sigma_i}{k_i},1\right)_{1,r},(1-\rho,r),(1-\rho-\alpha-\alpha'-\alpha''-\beta+\gamma,r),(1-\rho-\alpha'-\alpha''+\beta'\beta'',r)\\ \left(\frac{\vartheta_i}{k_i},\frac{\theta_i}{k_i},\frac{\mu_i}{k_i}\right)_{1,r},(1-\rho-\alpha-\alpha'-\alpha''+\gamma,r),(1-\rho-\alpha''+\alpha'-\beta+\gamma,r),(1-\rho+\beta',\beta'',r)\end{matrix}\middle|\frac{(s_ip_i)^{\left(1-\frac{\theta_i}{k_i}\right)}}{t^r}\right]\right\}\quad(3.4)$$

**Proof.** By utilising equation (1.6) and after re-arranging the order of integration and summation, we get (say $F_4$)

$$F_4=\prod_{i=1}^{r}\left\{\sum_{n_i=0}^{\infty}\frac{p_i{}^{(\sigma_i)}{}_{n_i,k_i,s_i}}{p_i{}^{\Gamma_{s_i,k_i}(n_i\theta_i+\vartheta_i)}}\frac{1}{n_i!}\right\}\left(D_{0+}^{\alpha,\alpha',\alpha'',\beta,\beta',\beta'',\gamma}t^{n_ir+\rho-1}\right)(t)\qquad(3.5)$$

Now, with the help of result (3.1) into (3.5) to obtain

$$F_4=\prod_{i=1}^{r}\left\{\sum_{n_i=0}^{\infty}\frac{p_i{}^{(\sigma_i)}{}_{n_i,k_i,s_i}}{p_i{}^{\Gamma_{s_i,k_i}(n_i\theta_i+\vartheta_i)}}\frac{1}{n_i!}\frac{\Gamma(1-\rho+n_ir)}{\Gamma(1-\rho+n_ir-\alpha-\alpha'-\alpha''+\gamma)}\right.$$

$$\left.\times\frac{\Gamma(1-\rho+n_ir+\gamma-\alpha-\alpha'-\alpha''-\beta)\Gamma(1-\rho+n_ir-\alpha'-\alpha''+\beta')\Gamma(1-\rho+n_ir-\alpha'-\alpha''+\beta'')}{\Gamma(1-\rho+n_ir+\gamma-\alpha'-\alpha''-\beta)\Gamma(1-\rho+n_ir+\beta')\Gamma(1-\rho+n_ir+\beta'')}t^{\rho+n_ir+\alpha+\alpha'+\alpha''-\gamma-1}\right\}$$

(3.6)

By includind the result of (1.7) and (1.8) into (3.6), we get

$$\mathfrak{J}_4=x^{\rho-\gamma+\alpha+\alpha'+\alpha''-1}\prod_{i=1}^{r}\left\{\frac{k_i(s_ip_i)^{-\vartheta_i/k_i}}{\Gamma\left(\frac{\sigma_i}{k_i}\right)}\sum_{n_i=0}^{\infty}\frac{1}{n_i!}\frac{\Gamma\left(\frac{\sigma_i}{k_i}+n_i\right)}{\Gamma((n_i\theta_i+\vartheta_i)/k_i)}\frac{\Gamma(1-\rho+n_ir)}{\Gamma(1-\rho+n_ir+\beta')}\right.$$

$$\left.\times\frac{\Gamma(1-\rho+n_ir-\alpha-\alpha'-\alpha''-\beta+\gamma)\Gamma(1-\rho+n_ir-\alpha'-\alpha''+\beta')\Gamma(1-\rho+n_ir-\alpha'-\alpha''+\beta'')}{\Gamma(1-\rho+n_ir-\alpha-\alpha'-\alpha''+\gamma)\Gamma(1-\rho+n_ir-\alpha'-\alpha''-\beta+\gamma)\Gamma(1-\rho+n_ir-\alpha'-\alpha''-\beta'+\gamma)}\right\}$$

$$\left((s_ip_i)^{\left(1-\frac{\theta_i}{k_i}\right)}t^r\right)^{n_i}$$

Finally, we obtained at the right hand side of result (3.4) after the re-insertion of the above series, which was in the form of generalized Wright hypergeometric function defined by (1.11).

**Corollary 4.** In the light of (1.10), then we get the new result in terms of Fox H-function:

$$\left(D_{-}^{\alpha,\alpha',\alpha'',\beta,\beta',\beta'',\gamma}t^{\rho-1}\left\{\prod_{i=1}^{r}p_iE_{k_i,\theta_i,\vartheta_i}^{\sigma_i,s_i}\left(\frac{1}{t}\right)\right\}\right)(t)=t^{\rho+\alpha+\alpha'+\alpha''-\gamma-1}\prod_{i=1}^{r}\left\{\frac{k_i(s_ip_i)^{-\frac{\vartheta_i}{k_i}}}{\Gamma\left(\frac{\sigma_i}{k_i}\right)}\right.$$

**Jilin Daxue Xuebao (Gongxueban)/Journal of Jilin University (Engineering and Technology Edition)**
ISSN: 1671-5497
E-Publication: Online Open Access
Vol: 41 Issue: 12-2022
DOI 10.17605/OSF.IO/U3TA7

$$\times H^{1,r+3}_{r+3,r+4}\left[\begin{array}{c|c}\left(1-\frac{\sigma_i}{k_i},1\right)_{1,r},(\rho,r)\,(\rho+\alpha+\alpha'+\alpha''+\beta-\gamma,r)(\rho+\alpha'+\alpha''-\beta',r)(\rho+\alpha'+\alpha''-\beta'',r) & \\ (0,1),\left(1-\frac{\vartheta_i}{k_i},\frac{\theta_i}{k_i},\frac{\mu_i}{k_i}\right)_{1,r},(\rho+\alpha+\alpha'+\alpha''-\gamma,r)\,(\rho+\alpha'+\alpha''+\beta-\gamma,r)(\rho-\beta',r)(\rho-\beta'',r) & \end{array} -\frac{(s_ip_i)^{\left(1-\frac{\theta_i}{k_i}\right)}}{t^r}\right]\Bigg\}.$$

## 4. Some special cases

We develop some special cases results connected to Saigo-Maeda fractional integral and differential operators, Saigo operators, Riemann-Liouville, Erdélyi – Kober operators etc .

**Corollary 5.** Let the condition of Theorem 1 be satisfied with $r=1,\ p_1=p,\sigma_1=\sigma,s_1=s,k_1=k,\theta_1=\theta$ and $\vartheta_1=\vartheta$ we get:

$$\left(I^{\alpha,\alpha',\alpha'',\beta,\beta',\beta'',\gamma}_{0+}t^{\rho-1}\{pE^{\sigma,s}_{k,\theta,\vartheta}(t)\}\right)(t)=t^{\rho+\gamma-\alpha-\alpha'-\alpha''-1}\left\{\frac{k(sp)^{-\frac{\vartheta}{k}}}{\Gamma\left(\frac{\sigma}{k}\right)}\right.$$

$$\times\,{}_4\Psi_4\left[\begin{array}{c|c}\left(\frac{\sigma}{k},1\right),(\rho,1),(\rho-\alpha'-\alpha''+\beta'+\beta'',1),(\rho+\gamma-\alpha-\alpha'-\alpha''-\beta,1) & \\ \left(\frac{\vartheta}{k},\frac{\theta}{k},\frac{\mu}{k}\right),(\rho+\beta',1),(\rho+\gamma-\alpha-\alpha'-\alpha'',1),(\rho+\gamma-\alpha'-\alpha''-\beta,1) & \end{array}(sp)^{1-\frac{\vartheta}{k}}\ \ t\right]\Bigg\}$$

(4.1)

In above equation (4.1), if we take p = s = k = 1 we obtain similar results as provided by Chouhan et. al. (2014).

**Corollary 6.** Let the condition of equation (2.1) be satisfied $\alpha$ is replaced by $\alpha+\beta,\alpha'=\alpha''=\beta'=\beta''=0,\ \ \beta=-\eta$ and $\gamma=\alpha$ in (2.1), we get the new result -concerning fractional integral equation:

$$\left(I^{\alpha,\beta,\eta}_{0+}t^{\rho-1}\left\{\prod_{i=1}^{r}p_iE^{\sigma_i,s_i}_{k_i,\theta_i,\vartheta_i}(t)\right\}\right)(t)$$

$$=t^{\rho-\beta-1}\prod_{i=1}^{r}\left\{\frac{k_i(s_ip_i)^{\frac{-\vartheta_i}{k_i}}}{\Gamma\left(\frac{\sigma_i}{k_i}\right)}\ {}_{r+2}\Psi_{r+2}\left[\begin{array}{c|c}\left(\frac{\sigma_i}{k_i},1\right)_{1,r},(\rho,r),(\rho+\eta-\beta,r) & \\ \left(\frac{\vartheta_i}{k_i},\frac{\theta_i}{k_i},\frac{\mu_i}{k_i}\right)_{1,r},(\rho-\beta,r),(\rho+\alpha+\eta,r) & \end{array}(s_ip_i)^{\left(1-\frac{\theta_i}{k_i}\right)}\ t^r\right]\right\} \qquad (4.2)$$

We correspond with the similar known results of Kabra and Nagar (2019).In above equation (4.2), if we take $r=1,\ p_1=p,\sigma_1=\sigma,s_1=s,k_1=k,\theta_1=\theta$ and $\vartheta_1=\vartheta$.

The known result of Ahmed (2014) agree with the above equation (4.2), if we take $r=1$, $p_1=p=1,\sigma_1=\sigma,s_1=s=1,k_1=k=1,\sigma_1=\sigma,\theta_1=\theta$ and $\vartheta_1=\vartheta$.

If $\beta$ is replaced by – $\alpha$ the equation (4.2), we achieve results regarding Riemann-Liouville fractional integral operator.

In the equation (4.2), if we take $\beta=0$ we obtain results in connection with Erdêlyi- Kober fractional integral operator.

**Corollary 7.** Let the condition of equation (4.5) be satisfied with $r=1,\ p_1=p,\sigma_1=\sigma,s_1=s,k_1=k,\theta_1=\theta$ and $\vartheta_1=\vartheta$ we get

$$\left(I_{-}^{\alpha,\alpha',\alpha'',\beta,\beta',\beta'',\gamma} t^{\rho-1}\left\{pE_{k,\theta,\vartheta}^{\sigma,s}(t)\right\}\right)(t) = t^{\rho-\alpha-\alpha'-\alpha''+\gamma-1} \frac{k(sp)^{-\frac{\vartheta}{k}}}{\Gamma\left(\frac{\sigma}{k}\right)}$$

$$\times\ {}_4\Psi_4\left[\begin{matrix} \left(\frac{\sigma}{k},1\right)_{1,r},(1-\rho-\beta,1),(1-\rho+\alpha++\alpha''\alpha'-\gamma,1),(1-\rho+\alpha+\alpha''+\beta'-\gamma,1) \\ \left(\frac{\vartheta}{k},\frac{\theta}{k},\frac{\mu}{k}\right)_{1,r},(1-\rho,1),(1-\rho+\alpha+\alpha'+\alpha''+\beta'-\beta''-\gamma,1),(1-\rho+\alpha-\beta,1) \end{matrix}\middle| \frac{(sp)^{\left(1-\frac{\vartheta}{k}\right)}}{t}\right] \quad (4.3)$$

In above equation (4.3), on putting p = s = k = 1 we get similar results as provided by chouhan et. al. (2014).

## CONCLUSION

In the present paper, we have develop the Mittag-Leffler function of Fractional Differential operators with the Appell's function as a kernel. Also develop some new type of fractional integral equations of Mittag-Leffler as special case, if we put β= -α and β=0 in equations (1.12), (1.13), (1.14) and (1.15).

### References

1) Ahmed S. (2014), on the generalized fractional integrals of the generalized Mittag-Leffler function, Springer Plus, doi: 10.1186/2193-1801-3-198.

2) Ayub,U., Mubeen, S., Abdeljawad, T., Rahman G. and Nisar, K.S. (2020), The new Mittag-Leffler function and it's applications , J. of Mathematics, 1-8.

3) Cerutti, R.A., Luque, L.L. and Dorrego, G.A. (2017). On the (p-k) Mittag – Leffler function, Applied Mathematical Sciences, Vol. 11, and 2541-2566.

4) Chouhan A. et.al. (2014), On Generlized Mittag-Leffler Function and Fractional Operator, NCATAS, 1-4.

5) Deora, Y. and Banerji, P.K. (1993), Double Dirichlet average of function $e^x$ using fractional derivative, J. Fractional Calculus, Vol. 3, 81-86.

6) Diaz, R. and Pariguan, E. (2007)., On Hypergeometric function and k- pochhammer symbol, Div. Math. , Vol.15, no.2, 179-192.

7) Dorrego, G. A. and Cerutti, R.A. (2012), the k-Mittag-Leffler function, Int. J. Contemp. Math. Sciences, Vol.7, no.15, 705-716.

8) Gehlot, K.S. (2012), the generalized k-Mittag-Leffler function, Int. J.Contemp. Math. Sciences, Vol.7, no.45, 2213-2219.

9) Gehlot, K. S. (2014), Differential equation of k-Bessel function and its properties, Nonl. Analysis and Differential equation, Vol. 2, no.2, 61-67.

10) Gehlot K.S. (2018), the p-k Mittag-Leffler function, Palestine Journal of Mathematics, Vol.7, no.2, 628-632.

11) Gehlot, K. and Nantomah, K. (2018), (p-q-k) gamma and beta function and properties, Int. J. of Pure and Applied Mathematics, Vol.118, 519-526.

12) Mittag-Leffler, G.M.(1903), Sur la nouvellefunction $E_\theta(z)$ C.R. Acd. Sci. Paris Vol. 137, 554-558.

ISSN: 1671-5497
E-Publication: Online Open Access
Vol: 41 Issue: 12-2022
DOI 10.17605/OSF.IO/U3TA7

13) Marichev, O.and,Handbook I. (1983), of integral transform and higher transcendental Theory and algorithum tables, Ellis Horwood, Chichester [John Wiley and Sons], New York, 1983.

14) Mishra, V. N. , Suthar, D. L. and Purohit S. D.(2017) Marichev – Saigo – Maeda fractional calculus operators Srivastava polynomial and generalized Mittag-Leffler function: Cogent Mathematics ,1-13.

15) Prabhakar, T.R. (1971), A singular integral equation with a generalizedMittag-Leffler function in the kernel,Yokohama Math. J. Vol. 19, 7-15.

16) Purohit S. D. , Suthar D. L. and Kalla S. L. (2012) Marichev – Saigo – Maeda fractional integraction operators of the Bessel functions : LXVII 21-32.

17) Ram C. and Choudhary P.(2014), Pathway fractional integration operator as generalized k-Mittag-Leffler function in itskernel, Int. J. Math. Arch.Vol. 5 no. 11, 114-120.

18) Salim, T. O. (2009), Some properties relating to the generalized Mittag-Leffler function , Adv. Appl. Math., Vol. 4, 21- 30.

19) Wiman, A. (1905), Uber den fundamental Satz in der theorie der funktionen$E_\alpha$ (z), Acta Math. Vol.29, 191-201.

20) Watugala, G.K. (1993), The Sumudu transform: a new integral transform to solve differential equations and control engineering problems, Int. J. Math. Educ. Sci. Technol., Vol. 24, 35-43.